

MIXED BOUNDARY CONDITIONS IN EUCLIDEAN QUANTUM GRAVITY

Giampiero Esposito^{1,2} and Alexander Yu Kamenshchik³

¹*Istituto Nazionale di Fisica Nucleare, Sezione di Napoli
Mostra d'Oltremare Padiglione 20, 80125 Napoli, Italy;*

²*Dipartimento di Scienze Fisiche
Mostra d'Oltremare Padiglione 19, 80125 Napoli, Italy;*

³*Nuclear Safety Institute, Russian Academy of Sciences
Bolshaya Tuskaya 52, Moscow 113191, Russia.*

Abstract. This paper studies a new set of mixed boundary conditions in Euclidean quantum gravity. These involve, in particular, Robin boundary conditions on the perturbed 3-metric and hence lead, by gauge invariance, to Robin conditions on the whole ghost 1-form. The corresponding trace anomaly is evaluated in the case of flat Euclidean 4-space bounded by a 3-sphere. In general, this anomaly differs from the ones resulting from other local or non-local boundary conditions studied in the recent literature.

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1. Introduction

Since the main problem in quantum cosmology is to find a suitable set of boundary conditions which completely determine the path integral for the quantum state of the universe [1-2], the search for such boundary conditions is of crucial importance in Euclidean quantum gravity [3]. For this purpose, it has been found in [4] that, following earlier work in [5], a set of gauge-invariant boundary conditions for Euclidean quantum gravity can be written as

$$\left[h_{ij} \right]_{\partial M} = 0 \quad (1.1)$$

$$\left[\frac{\partial h_{00}}{\partial \tau} + \frac{6}{\tau} h_{00} - \frac{\partial}{\partial \tau} (g^{ij} h_{ij}) + \frac{2}{\tau^2} h_{0i}{}^{||i} \right]_{\partial M} = 0 \quad (1.2)$$

$$\left[\frac{\partial h_{0i}}{\partial \tau} + \frac{3}{\tau} h_{0i} - \frac{1}{2} h_{00||i} \right]_{\partial M} = 0 \quad (1.3)$$

$$\left[\varphi_0 \right]_{\partial M} = 0 \quad (1.4)$$

$$\left[\varphi_i \right]_{\partial M} = 0. \quad (1.5)$$

With our notation [4], g is the flat background 4-metric, h is its perturbation, τ is the Euclidean-time coordinate, which becomes a radial coordinate if flat 4-space is bounded by a 3-sphere (see section 3). Moreover, φ_μ is the ghost 1-form, with normal component φ_0 and tangential components φ_i , and the stroke denotes covariant differentiation tangentially with respect to the 3-dimensional Levi-Civita connection of the boundary. We are interested

in the Faddeev-Popov formalism for quantum amplitudes. Thus, we deal with a gauge-averaging term in the Euclidean action, i.e.

$$I_{g.a.} = \frac{1}{32\pi G\alpha} \int_M \Phi_\nu \Phi^\nu \sqrt{\det g} \, d^4x \quad (1.6)$$

where Φ_ν is taken to be the de Donder functional (∇ being the 4-dimensional Levi-Civita connection of the background) [4,6]

$$\Phi_\nu^{dD}(h) \equiv \nabla^\mu \left(h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} h_{\rho\sigma} \right) \quad (1.7)$$

and we consider a ghost η_μ and an anti-ghost $\bar{\eta}_\mu$. These ghosts reflect the invariance of the classical theory under *gauge* transformations of metric perturbations of the form

$$\hat{h}_{\mu\nu} \equiv h_{\mu\nu} + \frac{1}{2} \nabla_\mu \varphi_\nu + \frac{1}{2} \nabla_\nu \varphi_\mu. \quad (1.8)$$

The boundary conditions for φ_μ are the same as those for η_μ and $\bar{\eta}_\mu$ [7]. This property should not be surprising, since already in the simpler case of Euclidean Maxwell theory the ghost and anti-ghost obey the same boundary conditions imposed on the scalar field occurring in the gauge transformations of the potential (see Appendix A). It is therefore sufficient to consider φ_μ , and then multiply by -2 the resulting contribution to the amplitudes, since the ghosts are anti-commuting and complex-valued.

The boundary conditions (1.1) reflect a natural choice in the classical variational problem in general relativity, where one often fixes at the boundary the intrinsic 3-geometry

[2,8]. Since, denoting by K_{ij} the unperturbed extrinsic-curvature tensor of the boundary ($K_{ij} \equiv \frac{1}{2} \frac{\partial g_{ij}}{\partial \tau} = \tau c_{ij} = \frac{1}{\tau} g_{ij}$, where c_{ij} is the metric on a unit 3-sphere), one has

$$\hat{h}_{ij} \equiv h_{ij} + \frac{1}{2} \varphi_{i|j} + \frac{1}{2} \varphi_{j|i} + K_{ij} \varphi_0 \quad (1.9)$$

the invariance of (1.1) under (1.9) is guaranteed providing (1.4)-(1.5) hold. Strictly, one has to require that $\left[\varphi_{i|j} \right]_{\partial M} = 0$, but this is automatically satisfied by (1.5), at least in the case of a 3-sphere boundary. The remaining boundary conditions (1.2)-(1.3) are obtained by setting to zero at the boundary the de Donder functional defined in (1.7). Their invariance under (1.8) is guaranteed providing (1.4)-(1.5) hold.

Note that the boundary conditions (1.1)-(1.5) are non-local in that they cannot be written in terms of complementary projection operators, and can instead be re-expressed as integral equations on metric perturbations at the boundary. The aim of our paper is to study in detail another relevant choice of mixed boundary conditions in Euclidean quantum gravity. The motivations of our analysis are as follows.

(i) In classical general relativity one can also fix at the boundary the trace of the extrinsic-curvature tensor (rather than the induced 3-metric) [8]. This choice of boundary conditions is also relevant for the theory of the quantum state of the universe [1,9].

(ii) In one-loop quantum cosmology, one can also consider Hawking's magnetic boundary conditions for quantum gravity, which set to zero at the boundary the linearized magnetic curvature, and hence the first derivatives with respect to τ of the perturbed 3-metric (see section 7.3 of [2]).

(iii) The boundary conditions (1.1)-(1.5) are gauge-invariant but non-local. By contrast, the Luckock-Moss-Poletti boundary conditions studied in [4,6,7,10], i.e.

$$\left[h_{ij} \right]_{\partial M} = 0 \quad (1.10)$$

$$\left[h_{0i} \right]_{\partial M} = 0 \quad (1.11)$$

$$\left[\frac{\partial h_{00}}{\partial \tau} + \frac{6}{\tau} h_{00} - \frac{\partial}{\partial \tau} (g^{ij} h_{ij}) \right]_{\partial M} = 0 \quad (1.12)$$

$$\left[\varphi_0 \right]_{\partial M} = 0 \quad (1.13)$$

$$\left[\frac{\partial \varphi_i}{\partial \tau} - \frac{2}{\tau} \varphi_i \right]_{\partial M} = 0 \quad (1.14)$$

are local but not entirely gauge-invariant, since (1.14) makes it impossible to preserve (1.10) under the action of (1.9). It therefore appears that neither locality nor complete gauge invariance of the boundary conditions are always respected in Euclidean quantum gravity. The minimal requirement is instead that the boundary conditions on the ghost 1-form φ_μ should lead to gauge invariance of at least a subset of the boundary conditions on metric perturbations. The latter choice reflects a careful analysis of well-posed variational problems in classical general relativity, which usually fix at the boundary the 3-metric, or its normal derivatives, or the trace of the extrinsic-curvature tensor [8].

Section 2 derives in detail our new set of mixed boundary conditions for the linearized gravitational field. Section 3 evaluates the corresponding trace anomaly by using zeta-function regularization. Concluding remarks and open problems are presented in section

4. Relevant details on boundary conditions and $\zeta(0)$ calculations are described in the appendices.

2. Mixed boundary conditions

For a given choice of quantization technique and background 4-geometry, the amplitudes of quantum gravity may depend on the boundary 3-geometry and on the boundary conditions, and hence a large number of interesting problems can be studied for various gauge-averaging terms in the Euclidean action. For the reasons described in the introduction, we are interested in the most general class of Robin boundary conditions on perturbations of the 3-geometry. Thus, denoting by λ a real dimensionless parameter, we begin by requiring that

$$\left[\frac{\partial h_{ij}}{\partial \tau} + \frac{\lambda}{\tau} h_{ij} \right]_{\partial M} = 0. \quad (2.1)$$

The invariance of the boundary conditions (2.1) under the transformations (1.9) is ensured by the following set of Robin conditions on the ghost 1-form:

$$\left[\frac{\partial \varphi_0}{\partial \tau} + \frac{(\lambda + 1)}{\tau} \varphi_0 \right]_{\partial M} = 0 \quad (2.2)$$

$$\left[\frac{\partial \varphi_i}{\partial \tau} + \frac{\lambda}{\tau} \varphi_i \right]_{\partial M} = 0. \quad (2.3)$$

Note that (2.3) is a local equation which ensures the validity of a more complicated condition, involving its 3-dimensional covariant derivative with respect to the Levi-Civita connection of the boundary.

At this stage, the remaining set of boundary conditions on metric perturbations can be obtained by bearing in mind that, in the case of flat Euclidean 4-space with a 3-sphere boundary, one has from (1.8)

$$\hat{h}_{00} = h_{00} + \frac{\partial \varphi_0}{\partial \tau} \quad (2.4)$$

$$\hat{h}_{0i} = h_{0i} + \frac{\partial \varphi_i}{\partial \tau} - \frac{2}{\tau} \varphi_i. \quad (2.5)$$

Thus, if we require

$$\left[h_{00} \right]_{\partial M} = 0 \quad (2.6)$$

this boundary condition is gauge-invariant providing one sets $\lambda = -1$ in (2.2), and if we require

$$\left[h_{0i} \right]_{\partial M} = 0 \quad (2.7)$$

this boundary condition is gauge-invariant providing one sets $\lambda = -2$ in (2.3). Of course, the whole set of boundary conditions (2.1)-(2.3), (2.6)-(2.7) does not involve complementary projection operators and is not gauge-invariant. Nevertheless, they seem to correspond to the most general form of quantum boundary-value problem which admits as a particular case (for $\lambda = 0$) the boundary conditions of section 7.3 of [2] on transverse-traceless perturbations, and such that the boundary conditions on the perturbed 3-geometry are always gauge-invariant. If λ vanishes, they also provide a generalization to pure gravity of the electric boundary conditions for Euclidean Maxwell theory, whilst the Barvinsky boundary conditions (1.1)-(1.5) generalize the magnetic boundary conditions for Maxwell theory (see appendix A).

3. $\zeta(0)$ value

The boundary conditions (2.1)-(2.3) and (2.6)-(2.7) are now applied to evaluate the trace anomaly in quantum cosmology, in the case of flat Euclidean 4-space bounded by a 3-sphere [2,4,10,11]. This may be regarded as the first step towards the analysis of curved backgrounds with boundary, or as a case relevant for quantum cosmology in the limit of small 3-geometries [11].

The trace anomaly is expressed through the value at the origin of the generalized zeta-function [12] obtained from the eigenvalues of the elliptic operators acting on metric perturbations and ghosts. Since the necessary formalism is developed and applied in detail in [4,6,13,14], we limit ourselves to a brief outline of the lengthy calculations involved in our analysis. First, one expands on a family of 3-spheres centred on the origin the metric perturbations h_{00}, h_{0i}, h_{ij} and the components φ_0, φ_i of the ghost 1-form. One then finds 10 sets of perturbative modes, 7 for $h_{\mu\nu}$ and 3 for φ_μ , which give rise to 8 contributions to $\zeta(0)$ resulting from transverse-traceless modes, scalar-type perturbations, vectorlike perturbations, a finite number of scalar modes, a decoupled vector mode, plus scalar ghost modes, vector ghost modes and a decoupled ghost mode. Decoupled modes belong to finite-dimensional subspaces, and multiply scalar harmonics, or longitudinal harmonics, or transverse harmonics.

The contributions of all modes to $\zeta(0)$ are obtained by combining their uniform asymptotic expansions as both the eigenvalues and the order tend to ∞ (the modes being linear combinations of Bessel functions) with their limiting behaviour as the eigenvalues tend to

0 and the order tends to ∞ . In the light of the boundary conditions of section 2, the 8 contributions to $\zeta(0)$ are found to be (see appendix B)

$$\zeta(0)_{\text{transverse-traceless modes}} = \frac{112}{45} + 3\lambda - \lambda^2 - \frac{1}{3}\lambda^3 \quad (3.1)$$

$$\zeta(0)_{\text{scalar modes}} = \frac{824}{45} - 2\lambda - \lambda^2 - \frac{1}{3}\lambda^3 \quad (3.2)$$

$$\zeta(0)_{\text{vector modes}} = \frac{434}{45} - 2\lambda - \lambda^2 - \frac{1}{3}\lambda^3 \quad (3.3)$$

$$\zeta(0)_{\text{decoupled scalar modes}} = -19 \quad (3.4)$$

$$\zeta(0)_{\text{decoupled vector mode}} = -\frac{21}{2} \quad (3.5)$$

$$\zeta(0)_{\text{scalar ghost modes}} = -\frac{59}{45} + 2\lambda + \frac{2}{3}\lambda^3 \quad (3.6)$$

$$\zeta(0)_{\text{vector ghost modes}} = -\frac{13}{90} - 2\lambda + \frac{2}{3}\lambda^3 \quad (3.7)$$

$$\zeta(0)_{\text{decoupled ghost mode}} = \frac{3}{2}. \quad (3.8)$$

By virtue of (3.1)-(3.8), the full $\zeta(0)$ value is

$$\zeta_\lambda(0) = \frac{89}{90} - \lambda - 3\lambda^2 + \frac{1}{3}\lambda^3. \quad (3.9)$$

Note that, since such a $\zeta(0)$ has a cubic dependence on λ , a real value of λ (compatible with reality of the eigenvalues E_n resulting from self-adjointness) always exists such that our trace anomaly (3.9) can agree with the values found in [4] in the case of Luckock-Moss-Poletti boundary conditions: $\zeta_{\text{LMP}}(0) = -\frac{758}{45}$, or Barvinsky boundary conditions:

$\zeta_B(0) = -\frac{241}{90}$. More precisely, since a third-order algebraic equation with real coefficients always admits at least one real solution, we know a priori that the equations $\zeta_\lambda(0) = -\frac{758}{45}$, or $\zeta_\lambda(0) = -\frac{241}{90}$, are solved by at least one real value of λ . In the former case, one finds that only one real value of λ exists, and it lies in the open interval $]2, 3[$. In the latter case there are three real roots: $\lambda_1 = -3\sqrt{3} + 4$, $\lambda_2 = 3\sqrt{3} + 4$, $\lambda_3 = 1$. The remarkable agreement when $\lambda = 1$ with the trace anomaly resulting from the Barvinsky boundary conditions deserves further thinking.

Other relevant values of $\zeta_\lambda(0)$ are the ones corresponding to the gauge invariance of the boundary conditions (2.6) or (2.7). By virtue of (3.9) one finds

$$\zeta_{-1}(0) = -\frac{121}{90} \tag{3.10}$$

$$\zeta_{-2}(0) = -\frac{1051}{90}. \tag{3.11}$$

As far as we can see, our trace anomaly (3.9) and the ones obtained in [4] reflect three different sets of boundary conditions for Euclidean quantum gravity, i.e. (1.1)-(1.5), (1.10)-(1.14) and (2.1)-(2.3), (2.6)-(2.7). The experts may also find it useful to remark that, for $\lambda = 0$, the result (3.1) yields $\zeta_{TT}(0) = \frac{112}{45}$, which agrees with section 7.3 of [2], which used a less powerful algorithm.

4. Concluding remarks

Although local supersymmetry makes it necessary to study local boundary conditions involving complementary projection operators which act on fields or potentials [2,7,10], the

resulting boundary conditions for metric perturbations are not invariant under the whole set of transformations (1.8). In Euclidean quantum gravity one has thus a choice between local boundary conditions whose restrictions on the perturbed 3-metric are not gauge-invariant, or other local or non-local boundary conditions which restrict perturbations of the 3-geometry in a gauge-invariant way. Our paper has studied mixed boundary conditions in Euclidean quantum gravity which involve, for the perturbed 3-metric, the most general form of Robin boundary conditions. The results of our investigation are as follows.

First, the boundary conditions of section 2 correspond to the quantum boundary-value problem which admits as a particular case the boundary conditions on the linearized magnetic curvature studied in section 7.3 of [2], and such that the boundary conditions on the perturbed 3-metric are always gauge-invariant. Second, a detailed evaluation of one-loop amplitudes is indeed possible, and the resulting trace anomaly, given by (3.9), differs in general from the $\zeta(0)$ values obtained from other local or non-local boundary conditions studied in the literature [4-7,10]. Third, when the dimensionless parameter $\lambda = 1$, or $-3\sqrt{3} + 4$, or $3\sqrt{3} + 4$, the trace anomalies resulting from the boundary conditions (1.1)-(1.5) and (2.1)-(2.3), (2.6)-(2.7) turn out to agree. Hence there seems to be a non-trivial link between trace anomalies, when at least the boundary conditions on the perturbed 3-geometry are gauge-invariant.

The result (3.9) is a relevant example of how quantum amplitudes may depend on the boundary conditions. However, the geometric form of the corresponding asymptotic expansion of the heat kernel remains unknown (cf [4]). So far, only the local boundary conditions (1.10)-(1.14) make it possible to obtain both geometric and analytic formulae

for the trace anomaly. It also appears interesting to investigate the non-local nature of the one-loop effective action, which is a linear combination of $\zeta(0)$ and $\zeta'(0)$, in the case of local and non-local boundary conditions in Euclidean quantum gravity. This calculation appears possible for the first time, after the thorough analysis of scalar fields appearing in [15,16]. Moreover, it appears necessary to understand whether any set of local and gauge-invariant boundary conditions can be found in Euclidean quantum gravity when a 4-dimensional background geometry is analyzed. In the two-dimensional case, the author of [17] has found that for $R^2 + T^2$ gravity with an independent spin-connection, local and gauge-invariant boundary conditions do actually exist. A possible extension of such results to 4-dimensional Riemannian geometries with boundary would be of crucial importance to complete the analysis of mixed boundary conditions in Euclidean quantum gravity.

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Appendix A

The boundary conditions (1.1)-(1.5) are a particular case of a more general set of mixed boundary conditions for Euclidean quantum gravity [5]. The most general form of mixed boundary conditions which are completely invariant under the transformations (1.8) on metric perturbations is [4]

$$\left[h_{ij}\right]_{\partial M} = \left[\hat{h}_{ij}\right]_{\partial M} = 0 \quad (A.1)$$

$$\left[\Phi_\nu(h)\right]_{\partial M} = \left[\Phi_\nu(\hat{h})\right]_{\partial M} = 0 \quad (A.2)$$

$$\left[\varphi_\nu\right]_{\partial M} = 0 \quad (A.3)$$

where Φ_ν is an *arbitrary* gauge-averaging functional, and the conditions (A.3) result from (A.1)-(A.2) and ensure their validity. The boundary conditions (A.1)-(A.3) generalize to pure gravity the magnetic boundary conditions for Euclidean Maxwell theory [14], i.e.

$$\left[A_k\right]_{\partial M} = 0 \quad (A.4)$$

$$\left[\Phi(A)\right]_{\partial M} = 0 \quad (A.5)$$

$$\left[\varphi\right]_{\partial M} = 0 \quad (A.6)$$

where A_μ is the potential, Φ is an arbitrary gauge-averaging functional, and φ is the ghost 0-form.

The boundary conditions (2.1)-(2.3) and (2.6)-(2.7) are instead a generalization of the electric boundary conditions for Euclidean Maxwell theory [14], i.e.

$$\left[A_0 \right]_{\partial M} = 0 \quad (A.7)$$

$$\left[\frac{\partial A_k}{\partial \tau} \right]_{\partial M} = 0 \quad (A.8)$$

$$\left[\frac{\partial \varphi}{\partial \tau} \right]_{\partial M} = 0. \quad (A.9)$$

Note that (A.6) ensures the invariance of (A.4)-(A.5) under the gauge transformations

$$\hat{A}_\mu \equiv A_\mu + \partial_\mu \varphi \quad (A.10)$$

whilst (A.9) ensures the invariance of (A.7)-(A.8) under the action of (A.10).

Appendix B

Since the calculations leading to (3.1)-(3.8) are not straightforward, we give a brief outline of the most difficult, i.e. the contribution of scalar-type perturbations to the trace anomaly. To avoid repeating ourselves, we refer the reader to [6,13,14] for the meaning of our notation and for the technique used in our investigation.

As the eigenvalues tend to ∞ and the order n of basis functions tends to ∞ , the n -dependent coefficient in the equation obeyed by the eigenvalues by virtue of boundary conditions takes the form (cf [4])

$$\rho_\infty(n) = 12 \frac{n(n^2 - 1)}{(n^2 - 4)}. \quad (B.1)$$

Thus, since the degeneracy of coupled scalar modes is n^2 for all $n \geq 3$, one has to consider $\frac{1}{2}n^2 \log(\rho_\infty(n))$, whose expansion as $n \rightarrow \infty$ does not have terms proportional to $\frac{1}{n}$. Hence $I_{\text{pole}}(\infty)$ vanishes. Moreover, as the eigenvalues tend to 0 whilst $n \rightarrow \infty$, the term contributing to $I_{\text{pole}}(0)$ in the eigenvalue condition reduces to (cf [4])

$$\rho_0(n) = \Gamma^{-4}(n) \left(1 - \frac{1}{n}\right) \frac{(n^2 + 1)}{n(n+1)(n+2)} \left[1 + 2 \frac{(\lambda + 1)}{n} f_1(n) + (\lambda + 1)^2 f_2(n)\right] \quad (B.2)$$

where

$$f_1(n) \equiv \frac{(n^4 - 2n^2 - 1)}{(n^2 + 1)(n^2 - 4)} \quad (B.3)$$

$$f_2(n) \equiv \frac{(n^2 - 1)}{(n^2 - 4)} \frac{1}{(n^2 + 1)}. \quad (B.4)$$

Thus, as $n \rightarrow \infty$, $\frac{1}{2}n^2 \log(\rho_0(n))$ has many contributions proportional to $\frac{1}{n}$, so that $I_{\text{pole}}(0)$ is found to be

$$I_{\text{pole}}(0) = -\frac{119}{180} + \lambda + \frac{1}{3}(\lambda + 1)^3. \quad (B.5)$$

Last, I_{\log} is obtained after eliminating fake roots in the eigenvalue condition, and then using the uniform asymptotic expansion of Bessel functions. This leads to

$$I_{\log} = 18 - \frac{1}{60}. \quad (B.6)$$

Since $\zeta(0) = I_{\log} + I_{\text{pole}}(\infty) - I_{\text{pole}}(0)$, equations (B.5)-(B.6) lead to the result (3.2).

References

- [1] Hawking S W 1984 *Nucl. Phys. B* **239** 257
- [2] Esposito G 1994 *Quantum Gravity, Quantum Cosmology and Lorentzian Geometries* (Lecture Notes in Physics **m12**) (Berlin: Springer)
- [3] Hawking S W and Gibbons G W 1993 *Euclidean Quantum Gravity* (Singapore: World Scientific)
- [4] Esposito G, Kamenshchik A Yu, Mishakov I V and Pollifrone G *One-Loop Amplitudes in Euclidean Quantum Gravity* (DSF preprint 95/16)
- [5] Barvinsky A O 1987 *Phys. Lett.* **195B** 344
- [6] Esposito G, Kamenshchik A Yu, Mishakov I V and Pollifrone G 1994 *Phys. Rev. D* **50** 6329
- [7] Luckock H C 1991 *J. Math. Phys.* **32** 1755
- [8] York J W 1986 *Found. Phys.* **16** 249
- [9] Hartle J B and Hawking S W 1983 *Phys. Rev. D* **28** 2960
- [10] Moss I G and Poletti S 1990 *Nucl. Phys. B* **341** 155
- [11] Schleich K 1985 *Phys. Rev. D* **32** 1889
- [12] Hawking S W 1977 *Commun. Math. Phys.* **55** 133
- [13] Barvinsky A O, Kamenshchik A Yu and Karmazin I P 1992 *Ann. Phys., NY* **219** 201

- [14] Esposito G, Kamenshchik A Yu, Mishakov I V and Pollifrone G 1994 *Class. Quantum Grav.* **11** 2939
- [15] Dowker J S and Apps J S 1995 *Class. Quantum Grav.* **12** 1363
- [16] Bordag M, Geyer B, Kirsten K and Elizalde E *Zeta-Function Determinant of the Laplace Operator on the D-Dimensional Ball* (UB-ECM-PF preprint 95/10)
- [17] Vassilevich D V *On Gauge-Invariant Boundary Conditions for 2D Gravity with Dynamical Torsion* (TUW preprint 95/6)